**POSTPOSITIVISM AND MATHEMATICS EDUCATION**

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**Introduction**

The purpose of this paper is to explore the relationship between Postpositivism and mathematics education. This paper is organized firstly by describing Positivism and some issues with it that set the stage for the development of other learning theories, such as Postpositivism and Radical Constructivism. Second, I describe some important aspects of Postpositivism, such as its ontological and epistemological bases. Next, I compare Postpositivism and Radical Constructivism which includes their similarities and differences. Lastly, I conclude this paper by offering some further remarks on Postpositivism.

**Positivism**

Positivism[[1]](#footnote-2) was a prevailing perspective up until the early 1900s, when other perspectives about learning and knowledge were presented and discussed. For example, the works of Piaget (1896–1980), von Glasersfeld (1917–2010), and Vygotsky (1896–1934) began to gain traction during the 1900s. Positivism falls under realism (Stinson, 2020). Realism is that an individual’s knowledge is a representation of an ontological reality (this knowledge is sometimes referred to as “True” knowledge with a capital “T”). Another way others describe realism is that knowledge exists independently of an individual (Phillips, 1987). Because of this, Positivism is a foundationalist epistemology (Phillips & Burbules, 2000). Foundationalism refers perspectives that take certain knowledge to be true without doubt. Phillips and Burbules (2000) personifies this aspect of foundationalism by saying that the knowledge has *authority*. Challenging this knowledge is futile since knowledge is taken as a direct representation of an ontological reality and the knowledge is “securely established… [and has] a secure foundation” (Phillips & Burbules, 2000, p. 6). Knowledge, in this view, is taken as neutral (without biases) and is derived from experiences[[2]](#footnote-3). In other words, foundationalists take certain knowledge for granted based on our experiences with the world. However, foundationalists attempt to take the smallest possible facts from our experiences with the world as the foundation for building other knowledge.

A major issue with foundationalism is that certain foundational facts, derived or directly obtained from experiences, may be apparent for one person but not another. One person may think some fact is “trivial,” often said in mathematics, while another person may not think so. If there is a dispute about what small fact (about reality) is trivial, then there is an issue about what knowledge can be derived from that fact. The results of what one can say and know about reality can change. For example, Descartes was a foundationalist, and Phillips and Burbules (2000) stated, “…Descartes’s famous premise ‘I think, therefore I am’ has been shown to be more involved than he believed…. In other languages it is not even possible to say ‘I think’” (p. 15). Descartes used “I think” as a foundational fact for building other knowledge, but this foundational fact would not be apparent for those whose languages do not allow them to say “I think.” Phillips and Burbules (2000) called this issue *The Relativity of the “Light of Reason*.”

Another issue with Positivism is that evidence, which may include experiences or the foundations for knowledge, can be used to support multiple theories. If one is to have a theory (note the singularity of theory) supported by collecting evidence and running reliable experiments to determine facts about reality, then that person may proclaim facts (about reality). In fact, it wouldn’t be a theory, this is something we would call a natural law (or law of nature) in conventional modern-day terms. This natural law would be irrefutable, one cannot find evidence that this law is not true about the phenomena it describes, and repeatable, various evidence can be brought upon this law and the same result(s) will come from it. If supposed two theories are to describe reality, then they must either be different in some cases and, following Positivism principles, one of these must accurately describe reality or, if this is not the case, they must be the same. Said another way, if two Positivists have what they believe to be two different theories that accurately describe some aspect of one true reality, then at least one of their theories must be objectively wrong. It would likely require an infinite amount of evidence (something that cannot be done; how would one know that all such evidence has been found?) to come to adequately support only one theory. This issue is called *The Underdetermination of Theory by Evidence* (Phillips & Burbules, 2000).

The biggest issue for Positivism, and many other epistemologies, is to account for the problem of induction. Simply put, what is the basis for assuming that things in the future will happen or behave identically (or even similarly to) the way(s) they’ve happened or behaved in past experiences? Securing the strongest of foundations for conclusions involving the results of future experiences would be best achieved through deductive reasoning. For example, some ancient philosophers wished to go about describing knowledge and what one could possibly know deductively (Piaget & Garcia, 1989) but this was found to be riddled with problems including the problem of induction (Popper, 1962). However, future experiences cannot be determined based on the evidence at the moment for wanting to make these deductions.

Because of the problematic issues mentioned above and others, many theories about learning (or philosophies about learning (Ernest, 2006)) such as (Radical and Social) Constructivism, Sociocultural theories, Critical theories, and Postpositivism were offered as a replacement for Positivism. As a note, these learning theories will be referred to as philosophies of learning or learning philosophies hereafter to avoid confusion, described in the next section.

**Postpositivism**

Postpositivism is not a foundationalist epistemology since knowledge can always be refined, developed, or changed based on new experiences or observations. Regarding this point, Phillips (2000) stated quite well, “it simply means that it no longer can be claimed that there are any *absolutely authoritative foundations* upon which [knowledge] is based” (p. 102). Postpositivists are often stereotyped into exclusively using experiments to learn more, but this is not the case as observational studies, for example, can also be used (Phillips & Burbules, 2000). Related to this point, quantitative research is not reserved to a Postpositivist’s methods just as qualitative research is not reserved to a Radical Constructivist’s methods. Just as many other philosophies of learning utilize various methods for learning about ideas of interest or understanding students’ conceptions, so too can Postpositivists use them.

Karl Popper (1902–1994), considered to be the Father of Postpositivism, wrote extensively about ideas that contributed to Postpositivism. *The Logic of Scientific Discovery* (1959) and *Conjectures and Refutations* (1962) are two examples of texts Popper authored that discuss these Postpositivistic ideas. The big ideas of Postpositivism are *falsifiability,* that knowledge is *conjecture*, and *repetition* and *regularities* (Popper, 1962).

*Falsifiability* (or *refutability* or *testability*) is the characteristic of admitting, or being subjected to, attempts to find evidence that a proposed theory, or conjecture, about an object or phenomenon is false. Said another way, theories must permit themselves to be falsified with evidence, if a purported theory cannot be falsified, then it is not a theory.[[3]](#footnote-4) When one attempts to show that an object has certain properties or that a phenomenon will always occur, one tries to show that no counterexamples for it currently exist. In other words, there is no evidence to show that the proposed theory is false. In a way, this idea turns the idea of epistemology around (a sort of reorientation) because instead of finding what it is that does work (knowledge, in the traditional sense), one finds what does not work. A benefit of this method of falsifying is that it is a deductive argument: if when one finds evidence that a proposed theory is not true, then it logically follows that the theory cannot be true for what it asserts. As Popper (1962) said, one can certainly perform an *ad hoc* modification to resolve whatever issue may arise, but this only serves to devalue the strength of that theory. To strengthen a proposed theory, one needs to show that attempts to falsify it have been explored, and if the theory withstands attempts to falsify it, then the theory holds at least for the time being. Related to this, one should also invite, and not prevent, others from bringing their own methods and critiques to the theory to attempt to refute it because withstanding further attempts to refute it serve to strengthen the theory.

The idea that all knowledge is *conjecture* ties back to the removal of authoritative knowledge sources; everything can be questioned or tested in an attempt to *falsify* it. One’s knowledge, one’s conjectures or theories, are put forward into the world, with their own experiences and observations, where these conjectures either are falsified or withstand attempts to falsify it by oneself or others (Popper, 1962). In this case they contribute to one’s knowledge and possibly others’ knowledge. For example, when one reads mathematics education research articles, one examines or critiques it and, if the article withstands attempts to falsify it, then the ideas contained in the article may become part of one’s knowledge. So, it can be cited as something that either one knows or the community of mathematics education researchers (in that specific topic of research) knows.

Because all knowledge is conjecture, any knowledge used to conduct experiments or tests is also conjectural. In other words, individuals are fallible and so too are their methods (Popper, 1962). Because of this, the knowledge one has of the world must be assumed to be fundamentally flawed, but the theories that one presents for falsification allow one to move closer to understanding an objective reality. This idea that there is an ontological reality but individuals cannot know or be aware that they know some its characteristics is called *critical realism* (Guba, 1990). This idea that one’s knowledge of the world is fundamentally flawed relates to *The Relativity of the “Light of Reason,”* where the foundations for our knowledge come from experiences, but these foundations lead one to easily falsifiable theories. In addition to this, one has no way of verifying that whatever knowledge one has matches with the singular true ontological reality. The combination of falsifiability and that knowledge is conjecture is defined by various terms, for example, Cook (1985) called it *critical multiplism* and Popper (1962) used *critical* *rationalism*.

Given the multitude of issues and considerations above, a reasonable question is, “How do we come to know things?” Popper (1962) stated that induction is not a logically valid method for coming to have knowledge. He proposed an idea of r*epetition* and *regularities*, he stated:

Without waiting, passively, for repetitions to impress or impose regularities upon us, we actively try to impose regularities upon the world. We try to discover similarities in it, and to interpret it in terms of laws invented by us. Without waiting for premises we jump to conclusions. (pg. 60)

Repetition is related, again, to the idea of fundamentally flawed knowledge about the world. However, in Popper’s view, one seeks regularities to understand the world. These regularities that one takes as knowledge may be an incorrect conclusion about the world, and these regularities are subject to falsifiability by repetitions through evidence (various observations and experiences). When one seeks or assumes these regularities, evidence is brought upon a theory (knowledge) in hopes to support this regularity. If the regularity is not maintained, then it is modified (a weakened theory) or a new theory must be proposed. If the regularity is maintained, then one continues the repetition and attempts to find new evidence to test it to either refute or strengthen the theory. An example of this is commonly found in mathematics, mathematicians certainly seek regularities when formulating conjectures and establishing theorems. Calculations serves as a way to test the theory, but mathematicians bring about a strong attempt to refute the mathematical statement: the general but arbitrarily chosen element of the relevant set. The method for showing that this theory (the mathematical statement) is true is where the mathematician shows that there is no counterexample, that it cannot be falsified, by use of a deductive argument (a mathematical proof). This theory (the theorem) can then be repeated however many times, with the assumption that one operates with the appropriate axioms and domain, to observe and experience this regularity (the result of the theorem).

Do not be mistaken that when one’s conjectures are refuted, they are necessarily discarded. Instead, this simply means that the new evidence, in the form of experiences or observations, does not support the conjecture (the theory). The refuted theories may be discarded, but they can also be modified, and replaced by a stronger theory. Popper (1962) stated that this new stronger theory “should pass some new, and severe, test” (p. 242). However, the old theory still serves a purpose for identifying what the new theory failed to do; as Popper (1962) said, “this… is a result of the growth of knowledge—of the incorporation of what was new and problematic knowledge into background knowledge…” (p. 247). To be clear, one’s theories may still be used in the contexts and with the evidence that supports it. However, it is important to note that they may not be considered “good” theories.

As an example, students may use calculations to test whether a given mathematical statement is true (Brown, 2014; Harel & Sowder, 1998). When they encounter a case where it fails (often brought to his or her attention by a researcher or teacher), the student may either modify their theory (the method that calculating various cases shows that it is true; inductive reasoning) by performing *ad hoc* revisions such as stating that it is not true in these cases. Lakatos (1976) called this the *method of monster-barring*. In this case, the student does not discard their theory, instead they remove the cases that do not work with it; by doing this, their theory works in the context and with the evidence that supports it. However, this is not a “good” theory because mathematics educators do not recommend it (this method of argumentation) since it is not logically valid when the student is attempting to prove a statement whose referenced domain includes the monstrous elements to be barred. These *ad hoc* revisions can also become quite tedious depending on the mathematical statement because they are likely done case by case as they are encountered, not by generalizing and restricting the domain. Even in this case, the proposed statement is weakened when the domain is further restricted[[4]](#footnote-5). The student may also propose a new theory (and possibly a new method such as a mathematical proof) by attempting to show that the statement is true in the general, or other, case(s).

In the realm of mathematics, these case by case calculations do not prove a yet to be determined statement, but may help mathematicians understand it. Popper (1962) wrote:

Refutations have often been regarded as establishing the failure of a scientist, or at least of his theory…. [but] every refutation should be regarded as a great success…. Even if a new theory… should meet an early death, it should not be forgotten; rather its beauty should be remembered and history should record our gratitude to it. (p. 243)

I believe that this quote encapsulates many experiences of mathematicians who spend many hours contemplating problems and loathing their failures, but that this is all part of progress (Pólya, 1957).

**The “Post-” in Postpositivism**

Postpositivism and other perspectives such as Postmodernism (or Poststructuralism [Stinson, 2020]) have a few things in common. One thing is that they share “Post-” and this prefix operates quite similarly yet differently in each case. “Post-” refers to a shift in ontological and epistemological bases. For Postpositivism, the shift is from realism to critical realism and from foundationalism to nonfoundationalism. For Postmodernism, the shift is from realism to “anti-universal realism” (Stinson, 2020, p. 8) and from, again, foundationalism to nonfoundationalism (Stinson & Bullock, 2012). *Anti-universal realism* is described as “multiple minds, multiple subjects, and multiple knowledges that reflect different socio-historical, -cultural, and -political locations” (Stinson, 2020, p. 8).

In each case, the ontological and epistemological bases change, but each goes about it in a different way under a variety of different assumptions and viewpoints. Postpositivists use falsifiability, repetition, and regularities to establish themselves as a perspective while Postmodernists use “discursive formation of knowledge made in and through discursive events” (Stinson, 2020, pp. 9–10) similar to how humans operate. From this, it is clear that Postpositivism and Postmodernism share similarities yet differ in some ways.

**Interpreting the Field of Mathematics Education Research with Postpositivism**

How do philosophies of learning relate to Postpositivism and how do both of those relate to mathematics education research? Put simply, a philosophy of learning provides a bias that one, as a researcher, carries into the realm of conducting research. Researchers can certainly attempt to control for this bias to maintain neutrality, but it is no guarantee it will work (Phillips & Burbules, 2000). Those authors argued that these biases connect to *The Relativity of the “Light of Reason*” and *The Underdetermination of Theory by Evidence* because they may influence the interpretation of data or analyses which may make certain knowledge “obvious” or “trivial.” There may be multiple theories that support whatever data a researcher may find, interpret, and analyze. To clarify, I am not saying that these biases or perspectives are hinderances to conducting and disseminating research, I am saying that they influence aspects of it. It is as Phillips (1990) said, “the point is that these [perspectives]… serve as lenses, not as blinders” (p. 41). In other words, a philosophy of learning that is used by researchers does not prevent those researchers from discussing, critiquing, or examining others’ research that uses a different philosophy of learning.

For philosophies of learning, two things regularly happen in conducting and disseminating research in mathematics education that can interpreted from a Postpositivistic perspective. These are first, transparency of “theoretical” background or lenses and second, peer-reviews (Schwandt, 1990). For beginning, and indeed some seasoned researchers, it is often recommended that the “theoretical” background that either informs or guides the research be discussed at length. Researchers try to make it clear to the reviewers (referees) about how they are thinking about and conducting the research. Many journals are peer-reviewed in a way that is consistent with this Postpositivistic perspective. Just as importantly, research submitted for peer-review is put forth for possible *falsification*. Reviewers bring their own examinations and critiques, which may be influenced by their perspectives, for disseminating research in the peer-review process. They offer recommendations, critiques, or, if they find parts of the research problematic, refutations. Following this, the author(s) submitting the article for peer-review may either agree to and implement revisions or defend their theories (parts of the research that admit falsifiability) against refutations. These methods for examining and critiquing are part of the *critical tradition* (Popper, 1962). There are many mathematics education research communities that engage in this Postpositivistic framework for contributing to knowledge of students, teachers, and mathematics. I do not claim that these two criteria exhaust the ways that Postpositivism engages with mathematics education; only that these two criteria are more easily explained in a short manner and within the realm of mathematics education research (Phillips & Burbules, 2000).

To provide more concrete examples, consider two cases taken from Hackenberg (2010) and Piaget and Garcia (1989). In Hackenberg (2010), she stated, "Following Piaget… and von Glasersfeld…, I *view* *operations* as mental actions” (p. 385, emphasis added). Here, Hackenberg does not attempt to show that *operations are mental actions*, instead she put forward her view on what informed or guided the research. In contrast to views guiding the research, the theories Hackenberg generated about students’ ways of operating underwent analysis and admitted falsifiability because they were conjectural. She said, “Ongoing analysis occurs… to make *conjectures* about students’ cognitive constructs and ways of operating…, and to plan for future teaching episodes to *test those conjectures*” (p. 401, emphasis added). These new teaching episodes provide new experiences and observations that she uses to attempt to falsify her conjectures about the students’ understandings. I do not claim that Hackenberg is a Postpositivist or that her phrasings were intentional in their alignments with Postpositivism, I only point out these aspects of her work to show that Postpositivism can be used to interpret the way research is conducted, analyzed, and disseminated. Indeed, the research journal in which this article was published conducts peer-reviews for submissions and this research was “part of [her] dissertation” (Hackenberg, 2010, p. 429) where each was part of a series of examinations and critiques by other researchers consistent with the Postpositivistic idea of critical tradition.

In Piaget and Garcia’s (1989) *Psychogenesis and the History of Science*, they set out to argue that the development and progress of the sciences, including mathematics, follows the same progression as that of the mind of the child. Piaget is well-known for his contributions to (educational) psychology, but their book does not seek to prove Piaget’s perspectives, on the contrary, his perspectives are used to attempt to justify something else. The authors of that book do their best to justify this conjecture. For example, they said:

Our research will… be directed toward the identification of the mechanisms operating at each transition from one stage to the next in the evolution of concepts and theories. This will be done for each of the fields of science chosen to illustrate and to ‘verify’ *the fundamental hypotheses* that guide our overall conception. (p. 31, emphasis added)

In some cases, they anticipated possible refutations of their conjectures and defended their theory. Among these, a refutation they anticipated was “if at a certain age in the child, she is able to operate at these higher levels, why would these historical developments not occur earlier in history?” Piaget and Garcia (1989) answered this by mentioning *genetic assimiliation* and that “human intelligence is a special case of ‘behavior…’” (p. 139). In other words, humans are heavily influenced by their intellectual environment, so that if all members are operating at a certain level (according to Piaget and Garcia), then those children will learn these behaviors and processes as well. If they have indeed proven that the sciences progressed very similarly as it does in the child’s mind, then this would be quite a strong theory, having withstood many attempts to falsify it.

These two examples are both from a Radical Constructivist perspective, and I will discuss similarities and differences between Postpositivism and Radical Constructivism in the next section.

**Comparing Postpositivism and Radical Constructivism**

The guiding principles of Radical Constructivism are:

1. Knowledge is not passively received either through the senses or by way of communication. Knowledge is actively built up by the cognizing subject.
2. a. The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability;

b. Cognition serves the subject’s organization of the experiential world, not the discovery of an objective ontological reality. (von Glasersfeld, 1990, pp. 22–23)

The biggest difference between Postpositivism and Radical Constructivism is that the former is aligned with critical realism while the latter is aligned with *relativism*. Relativism is the idea that *realities* are based on a person’s experiences and observations for a person by mentally *constructing* *models* of his or her *experiential reality* (Guba, 1990; von Glasersfeld, 1990).

The idea of repetition and regularities is similar to the construction of models of the cognizing subject’s experiences and observations where this constitutes his or her experiential reality for the Radical Constructivist. von Glasersfeld (1990) would agree that an individual would construct and engage with a model for experiences that seemingly repeat themselves and this model would describe the regularities of the individual’s experiential reality.

Falsifiability may be incorporated by an individual whose models of his or her reality fails to account for a new experience. In other words, the individual may expect regularity based on their model, but the eventual result is not as expected. Radical Constructivists describe the individual’s resulting mental action as either *accommodation* or *assimilation* (von Glasersfeld, 1990). He described accommodation as “a change of model” (von Glasersfeld, 1988, p. 7), where model also represents knowledge, and assimilation as “treating new material as an instance of something known” (von Glasersfeld, 1995, p. 62). In fact, von Glasersfeld (1990) wrote, “…abstracting regularities and rules from experience that enable [one] to avoid disagreeable situations and to, some extent, to generate agreeable ones. This ‘abstracting of regularities’ is always the result of assimilation” (p. 24). This ties directly to repetition and regularities above; however, it may be the duty of the individual to intentionally attempt to falsify the model to better understand his or her reality.

As for the idea that knowledge is conjecture, I believe Radical Constructivists may find this palatable. All that one may know about one’s reality, based on those models which construct it, can change at any moment whenever a new experience occurs and the individual assimilates or accommodates it.

In all these comparisons, I feel that Radical Constructivism and Postpositivism are quite similar. However, a note about induction: Popper (1962) discussed a concern for epistemologies to resolve the issue of induction. He introduced his idea of *conjectures* and *refutations* which included the idea of falsifiability, conjectural knowledge, repetition, and regularities (see the section of Postpositivism above). I feel that Radical Constructivists account for this similarly but that one may note that Radical Constructivists often do not discuss this idea or issue of induction. It may be that the issue naturally resolves itself by definition of Radical Constructivists terms of models and experiential realities. An individual’s models of his or her world need not be logical, they must only work (von Glasersfeld, 1995).

However, from my perspective, neither Postpositivism nor Radical Constructivism provides a sufficient description for how young children, including infants, learn or come to know things about the world (or their worlds). Popper (1962) described that “…every organism has inborn *reactions* or *responses*; and among them, responses adapted to impending events” (p. 62). Piaget and Garcia (1989) discussed that children are born with certain characteristics from which their realities are constructed. They stated, “considering now the knowledge acquired before it gets formalized, we find at all levels the same twofold process in the constitution of instruments: comparative tools, built on correspondences, and transformational operations” (p. 11). I interpret this as they do not mean “find” (which is problematic in the Radical Constructivists perspective so it may have been lost in translation), and that this twofold process is a foundation for a child’s knowledge, they even go further to say, “…the two become increasingly interrelated, particularly since it has been possible to study transformations in themselves” (p. 11). The issue with these accounts, from my perspective, is that these rely on assumptions about humans—they rely on *foundations* from which their learning philosophies grow and develop. It was discussed earlier in this paper that both Postpositivism is nonfoundationalist and, in fact, Radical Constructivism is as well, but inevitably require these foundations to continue to say anything about the human phenomenon of learning.

Related to phenomena, Postpositivism accounts well for mathematics in the sense that one desires to identify regularities. Regardless of whether mathematics is a construction (as in Radical Constructivism) or not, in this Postpositivistic framework, mathematics is a regularity that lends itself to countless repetitions. I feel that Radical Constructivists find themselves in a much more difficult position since they must account for these multiple realities. On another note, Piaget and Garcia (1989) stated:

As for the successive and ultimate finalities of the process leading from the intra to the inter and then to the trans levels, the answer seems fairly obvious: neither the mathematician nor the child at a certain level are ever satisfied with simply observing and discovering (which means inventing things): at each stage, they strive to attain the “reasons” for what they find. This then comes down to a search for the “necessity” inherent in the generalities and relations established, since the subject does not accept, in the final analysis, a construction as being valid unless it acquires intrinsic necessity through explicit reasons. (p. 169)

In their book, they described that this theory was somewhat difficult to articulate and defend but nonetheless believed it enough to write a book about it. I interpret the quote above, when they discussed the development of algebra, as those authors defaulting to foundational characteristics of human desire to understand relationships in any way possible.

**Concluding Remarks**

The purpose of this paper was to analyze Postpositivism for mathematics education research, and to compare with other frameworks. I first began by describing Positivism and three issues that contributed to its demise, from where Postpositivism and other learning philosophies developed. I identified several important aspects of Postpositivism such as its ontological and epistemological bases that differentiate it from Positivism and relate it to more contemporary learning philosophies. Finally, I compared Postpositivism and Radical Constructivism.

An importance difference between Positivism and Postpositivism is that the former relies on foundationalism and realism while the latter doesn’t. The strongest similarity is that each relies on the assumption of an ontological reality but these are viewed quite differently.

Postpositivism introduces the important idea of falsifiability. One attempts to find what doesn’t work when engaging with the world. In this way, Postpositivism describes well how an individual comes to have knowledge. There are real problems with an individual coming to have knowledge for epistemologies that rely on foundationalism or realism (Popper, 1962; von Glasersfeld, 1984, 1990). Postpositivists release themselves from ontological constraints by stating that we cannot know the one true reality. One simply finds what cannot be in this world which is permitted by the one true reality.

In addition, the idea that knowledge is conjecture, or theory, fits with what contemporary philosophers describe as knowledge. Derived from the observation of repetition and regularities in our experiences, this differs from traditional ideas that knowledge should be deductive.

Finally, I agree with Simon (2009) that it is a mistake to believe that one philosophy of learning is better than another in the sense that it must replace it. One philosophy of learning is not a subset of another unless explicitly described that way. They are simply different perspectives on the human phenomenon of learning. In this Postpositivistic perspective, each philosophy of learning provides new theories that help us understand the world, our places in it, and the interactions between them and each other.

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1. Notably, there are at least two types: Comtean (or Classical) Positivism and Logical Positivism (Phillips, 1987). Generally, when one speaks of Positivism, they are likely unknowingly referring to the latter. However, the descriptions of those require more time and are beyond the scope of this paper, but nonetheless just as intriguing. [↑](#footnote-ref-2)
2. There are at least two perspectives that branch from foundationalism when discussing experiences: rationalism, relying on experiences, and empiricism, relying on sensory experiences (Phillips & Burbules, 2000). [↑](#footnote-ref-3)
3. This hopefully addresses the confusion that may have risen regarding learning theories and philosophies of learning. [↑](#footnote-ref-4)
4. This is sometimes done in mathematics but with different circumstances and purposes (Polya, 1957). [↑](#footnote-ref-5)